Sample Problems

#1 Analysis of Curves

1. (a) Find the critical numbers of **f** (if any), (b) find the open interval(s)non which the function is increasing and decreasing (c) and identify all relative extrema for the function $f(x) = x^2 - 6x$

2. Find the points of inflection and discuss the concavity of the graph of the function $f(x) = (1/4)x^4 - 2x^2$

3. Find all relative extrema, using the second derivative where necessary, for the function $f(x) = x^4 - 4x^3 + 2$

4. Determine whether he following sequences are monotonic
   a. $(n^2 - 1)/n$ b. $5 - (-1)^n$

#2 Planar Curves

Parametric

1. Sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter for the following equations
   $x = 3t - 1$ $y = 2t - 1$

2. For the following equations, find dy/dx and d^2y/dx^2, and find the slope and concavity (if possible) at the given value of the parameter.
   a) $x = 2t$ $y = 3t - 1$
   b) $x = 2 + \sec\theta$ $y = 1 + 2\tan\theta$

Polar

1. Find the corresponding rectangular coordinates for the point $(4, 3\pi/6)$
2. Find dy/dx and the slopes of the tangent lines of the polar equation
   $r = 2 + 3\sin\theta$
   at the points $(5, \pi/2)$, $(2, \pi)$, and $(-1, 3\pi/2)$

Vectors

1. Find the vectors **u** and **v** whose initial and terminal points are given. Show that **u** and **v** are equivalent.
   u: (3,2), (5,6)
   v: (-1, 4), (1,8)

2. Find (a) (2/3)**u**, (b) **v** – **u**, and (c) 2**u** + 5**v** for the vector equations
   u = ‹4, 9›   **v** = ‹2, -5›

#3 Optimization

1. A manufacturer wants to design an open box having a square base and surface area of 108 inches. What dimensions will produce a box with maximum volume?

2. A rectangular page in to contain 24 square inches of print. The margins at the top and bottom of the page are to be 3/2 inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

3. Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a singe stake, running from ground level to the top of each post. Where should the stake be placed to use the least amount of wire?

#4 Related Rates (Ch 2.6)

Equation	Find	Given
1. $y=\sqrt{x}$	a) dy/dt when x=4	dx/dt=3
	b) dx/dt when x=25	dy/dt=2
2. $y=2(x^2-3x)$	a) dy/dt when x=3	dx/dt=2
	b) dx/dt when x=1	dy/dt=5

A point is moving along the graph of the given function such that dx/dt is 2 centimeters per second. Find dy/dt for the given values of x.

3. $y=x^2 + 1$ a) x= -1   b) x=0
4. y=tanx   a) x= -π/3   b) x= -π/4

5. A spherical balloon is expanding. If the radius is increasing at the rate of 2 inches per minute, at what rate is the volume increasing when the radius is 5 inches?

6. At a sand and gravel plant, sand if falling off a conveyer and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 ft high?

7. An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other. One plane is 150 miles from the point moving at 450 mph. The other is 200 mi from the point moving at 600 mph.
   a) At what rate is the distance between the planes decreasing?

b) How much time does the air traffic controller have to get one of the planes on a different path?

8. A balloon leaves the ground 200 meters away from an observer and rises vertically at a rate of 50 meters per minute.

a) At what rate is the distance between the balloon and the observer increasing at the instant the balloon is 200 m above the ground?

b) At that same instant, at what rate is the angle of inclination of the observer's line of sight increasing?

#5 Using Implicit Differentiation to Find the Derivative of an Inverse Function (Ch 5.3)

Find dy/dx at the given point for the equation:

1. $x = y^4 - 2y^2 + 6$ (14, -2)
2. $x = 3y^3 - \sqrt{y}$ (2, 1)
3. $x = \tan y - 4$ (-3, π/4)
4. $x = 2\ln(y^2 - 3)$ (0, 4)

#4/6 Rate of Change (Ch 2.2)

Find the average rate of change of the function over the given interval. Compare this with the instantaneous rates of change at the endpoints of the interval.

1. $f(t) = 2t + 7$ [1,2]
2. $g(t) = t^2 - 3$ [7, 7.5]
3. $f(x) = -1/x$ [1,2]

Vertical Motion: (use the position function $s(t) = -16t^2 + V_0t + S_0$)

4. A silver dollar is dropped from the top of a building that is 1362 feet tall.

a) Determine the position and velocity functions for the coin.

b) Determine the average velocity on the interval [1, 2]

c) Find the instantaneous velocities when t =1 and t=2.

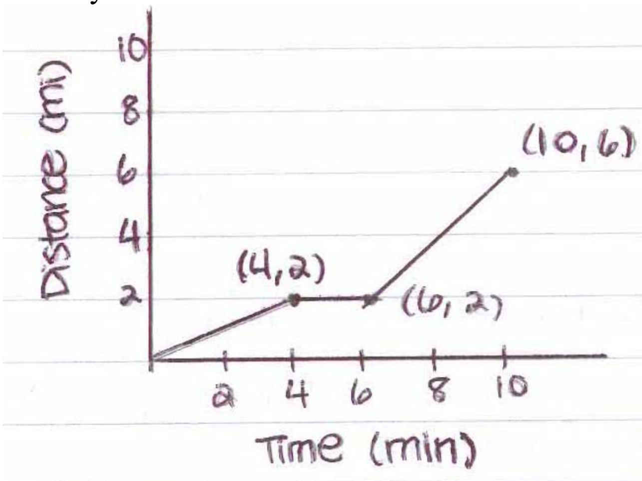
d) Find the time required for the coin to reach ground level.

e) Find the velocity of the coin at impact.

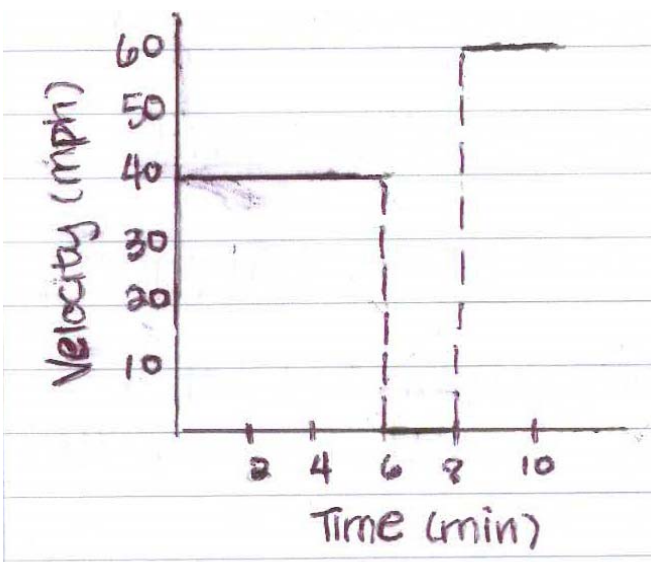
5. A car is driven 15,000 miles a year and gets X miles per gallon. Assume that the average fuel cost is $1.55 per gallon. Find the annual cost of fuel C as a function of X and use this function to complete the table.

X	10	15	20	25	30	35	40
C							
dC/dx							

6. The graph of a position function is shown. It represents the distance in miles that a person drives during a 10-minute trip to work. Make a sketch of the corresponding velocity function.



7. The graph of a velocity function is shown. It represents the velocity in miles per hour during a 10-minute drive to work. Make a sketch of the corresponding position function.

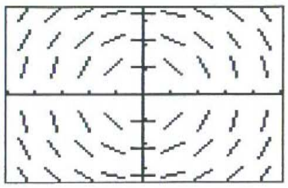


8. The volume of a cube with sides of length s is given by $V = s^3$. Find the rate of change of the volume with respect to s when s = 4 cm.

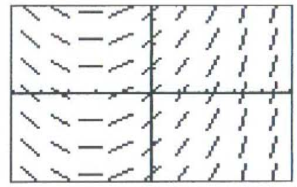
#7 Slope Fields

Match the slope fields with their differential equations.

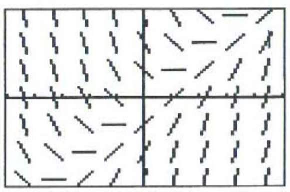
(A)



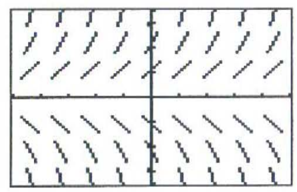
(B)



(C)



(D)



1. $\frac{dy}{dx} = \frac{1}{2}x + 1$

2. $\frac{dy}{dx} = x - y$

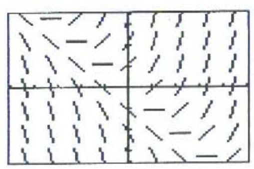
3. $\frac{dy}{dx} = y$

4. $\frac{dy}{dx} = -\frac{x}{y}$

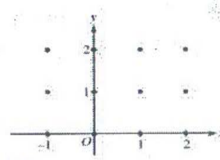
5. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.

(a) Sketch the solution curve through the point (0, 1).

(b) Sketch the solution curve through the point $(-3, 0)$.



6. Consider the differential equation $\frac{dy}{dx} = 2x - y$.



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0, 1)$. **(Note: Use the axes provided in the pink test booklet.)**

(b) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

#8 Euler's Method

(1) Use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use n steps of size h.

$y' = .5x(3-y)$, $y(0) = 1$, $n = 5$, $h = 0$

(2)

Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

(a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

#9 L'Hospital's Rule

(1) $\lim\limits_{x \to +\infty} \frac{\ln\left(x^3 + 2\right)}{\ln\left(5x^3 - 1\right)} =$

(2) $\lim\limits_{x \to +\infty} \frac{1/x^2}{\sin\left(1/x\right)} =$

(3) – Determine the convergence or divergence of the series:

$$\sum_{n=2}^{\infty} \left(\frac{\ln(n)}{n^3}\right)$$